

EXERCISE 5.3

In Problems 1–8, find exact solutions over the indicated intervals (x real and θ in degrees).

1. $2 \cos x + 1 = 0$, $0 \leq x < 2\pi$
2. $2 \sin x + 1 = 0$, $0 \leq x < 2\pi$
3. $2 \cos x + 1 = 0$, all real x
4. $2 \sin x + 1 = 0$, all real x
5. $\sqrt{2} \sin \theta - 1 = 0$, $0^\circ \leq \theta < 360^\circ$
6. $2 \cos \theta - \sqrt{3} = 0$, $0^\circ \leq \theta < 360^\circ$
7. $\sqrt{2} \sin \theta - 1 = 0$, all θ
8. $2 \cos \theta - \sqrt{3} = 0$, all θ

In Problems 9–14, solve each to four decimal places (x real and θ in degrees).

9. $4 \tan \theta + 15 = 0$, $0^\circ \leq \theta < 180^\circ$
10. $2 \tan \theta - 7 = 0$, $0^\circ \leq \theta < 180^\circ$

11. $5 \cos x - 2 = 0$, $0 \leq x < 2\pi$
12. $7 \cos x - 3 = 0$, $0 \leq x < 2\pi$
13. $5.0118 \sin x - 3.1105 = 0$, all real x
14. $1.3224 \sin x + 0.4732 = 0$, all real x

B For Problems 15–26, find exact solutions (x real and θ in degrees).

15. $\cos x = \cot x$, $0 \leq x < 2\pi$
16. $\tan x = -2 \sin x$, $0 \leq x < 2\pi$
17. $\cos^2 \theta = \frac{1}{2} \sin 2\theta$, all θ
18. $2 \sin^2 \theta + \sin 2\theta = 0$, all θ
19. $\tan(x/2) - 1 = 0$, $0 \leq x < 2\pi$
20. $\sec(x/2) + 2 = 0$, $0 \leq x < 2\pi$
21. $\sin^2 \theta + 2 \cos \theta = -2$, $0^\circ \leq \theta < 360^\circ$
22. $2 \cos^2 \theta + 3 \sin \theta = 0$, $0^\circ \leq \theta < 360^\circ$

23. $\cos 2\theta + \sin^2 \theta = 0$, $0^\circ \leq \theta < 360^\circ$
24. $\cos 2\theta + \cos \theta = 0$, $0^\circ \leq \theta < 360^\circ$
25. $4 \cos^2 2x - 4 \cos 2x + 1 = 0$, $0 \leq x \leq 2\pi$
26. $2 \sin^2(x/2) - 3 \sin(x/2) + 1 = 0$, $0 \leq x \leq 2\pi$

Solve Problems 27–30 (x real and θ in degrees). Compute inverse functions to four significant digits.

27. $4 \cos^2 \theta = 7 \cos \theta + 2$, $0^\circ \leq \theta \leq 180^\circ$
28. $6 \sin^2 \theta + 5 \sin \theta = 6$, $0^\circ \leq \theta \leq 90^\circ$
29. $\cos 2x + 10 \cos x = 5$, $0 \leq x < 2\pi$
30. $2 \sin x = \cos 2x$, $0 \leq x < 2\pi$

Solve Problems 31 and 32 for all real solutions. Compute inverse functions to four significant digits.

$$31. \cos^2 x = 3 - 5 \cos x \qquad 32. 2 \sin^2 x = 1 - 2 \sin x$$

33. Explain the difference between evaluating the expression $\cos^{-1}(-0.7334)$ and solving the equation $\cos x = -0.7334$.

34. Explain the difference between evaluating the expression $\tan^{-1}(-5.377)$ and solving the equation $\tan x = -5.377$.

C Find exact solutions to Problems 35–38. [Hint: Square both sides at an appropriate point, solve, then eliminate any extraneous solutions at the end.]

35. $\sin x + \cos x = 1$, $0 \leq x < 2\pi$
36. $\cos x - \sin x = 1$, $0 \leq x < 2\pi$
37. $\sec x + \tan x = 1$, $0 \leq x < 2\pi$
38. $\tan x - \sec x = 1$, $0 \leq x < 2\pi$

Exercise 5.3

1. $2\pi/3, 4\pi/3$
3. $2\pi/3 + 2k\pi, 4\pi/3 + 2k\pi, k$ any integer
5. $45^\circ, 135^\circ$
7. $45^\circ + k(360^\circ), 135^\circ + k(360^\circ), k$ any integer
9. 104.9314° 11. $1.1593, 5.1239$
13. $0.6696 + 2k\pi, 2.4720 + 2k\pi, k$ any integer
15. $\pi/2, 3\pi/2$
17. $90^\circ + k(180^\circ), 45^\circ + k(180^\circ), k$ any integer
19. $\pi/2$ 21. 180° 23. $90^\circ, 270^\circ$
25. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$ 27. 104.5°
29. $0.9987, 5.284$
31. $0.9987 + 2k\pi, -0.9987 + 2k\pi, k$ any integer
33. $\cos^{-1}(-0.7334)$ has exactly one value, 2.3941 ; the equation $\cos x = -0.7334$ has infinitely many solutions, which are found by adding $2\pi k, k$ any integer, to each solution in one period of $\cos x$.
35. $0, \pi/2$ 37. 0 39. 0.002613 sec 41. 33.21°
43. 64.1° 45. $(r, \theta) = (1, 30^\circ), (1, 150^\circ)$
47. $\theta = 45^\circ$

$$[2] \quad 2 \sin x + 1 = 0, \quad 0 \leq x \leq 2\pi$$

$$\sin x = -\frac{1}{2}$$

$$\therefore x = \frac{7\pi}{6} \text{ or } x = \frac{11\pi}{6}$$

NOTE that $-\frac{\pi}{6}$ is not an option since x required to be in $0 \leq x \leq 2\pi$

$$[4] \quad 2 \sin x + 1 = 0, \quad x \in \mathbb{R}$$

$$\therefore x = -\frac{\pi}{6} + 2n\pi, \quad x = \frac{7\pi}{6} + 2n\pi, \quad n \in \mathbb{Z}$$

$$[6] \quad 2 \cos \theta - \sqrt{3} = 0, \quad 0^\circ \leq \theta \leq 360^\circ$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ, \quad \theta = 330^\circ$$

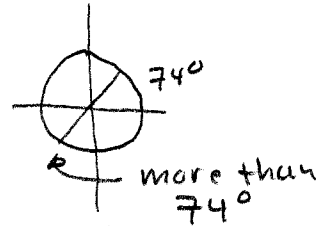
$$[8] \quad 2 \cos \theta - \sqrt{3} = 0, \quad \theta \in \mathbb{R}$$

$$\therefore \theta = 30^\circ + 360n, \quad \theta = 330^\circ + 360n$$

$$[10] \quad 2 \tan \theta - 7 = 0, \quad 0^\circ \leq \theta \leq 180^\circ$$

$$\tan \theta = \frac{7}{2}$$

$$\theta = \tan^{-1}\left(\frac{7}{2}\right) = 74.0546^\circ$$

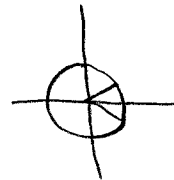


$$[12] \quad 7 \cos x - 3 = 0, \quad 0 \leq x \leq 2\pi$$

$$\cos x = \frac{3}{7}$$

$$x = \cos^{-1}\left(\frac{3}{7}\right)$$

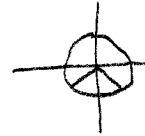
$$x = 1.1279, \quad 5.1553$$



$$2\pi - 1.1279 =$$

$$[14] \quad 1.3224 \sin x + 0.4732 = 0, \quad x \in \mathbb{R}$$

$$\sin x = \frac{-0.4732}{1.3224}$$



$$\therefore x = -0.3659 + 2n\pi \quad \text{or} \quad x = 3.5075 + 2\pi$$

$$[16] \quad \cos x = \cot x, \quad 0 \leq x \leq 2\pi$$

$$\cos x = \frac{\cos x}{\sin x}$$

$$\sin x \cos x = \cos x$$

$$\cos x (\sin x - 1) = 0$$

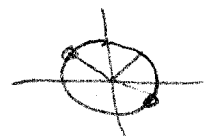
$$\cos x = 0 \quad \text{or} \quad \sin x = 1$$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$[18] \quad 2 \sin^2 \theta + \sin 2\theta = 0, \quad \text{all } \theta$$

$$\equiv 2 \sin^2 \theta + 2 \sin \theta \cos \theta = 0$$

$$\equiv 2 \sin \theta (\sin \theta + \cos \theta) = 0$$



$$\Rightarrow 2 \sin \theta = 0 \quad \text{or} \quad \sin \theta + \cos \theta = 0$$

$$\Rightarrow \theta = 360^\circ n \quad \text{or} \quad \sin \theta = -\cos \theta$$

$$\theta = 135^\circ, \theta = 315^\circ$$

$$\therefore \theta = 135^\circ + 360^\circ n \quad \text{or} \quad \theta = 315^\circ + 360^\circ n \quad \text{or} \quad \theta = 360^\circ n$$

$$[20] \quad \sec\left(\frac{x}{2}\right) + 2 = 0, \quad 0 \leq x \leq 2\pi$$

$$\sec\left(\frac{x}{2}\right) = -2$$

$$\sec\left(\frac{2\pi}{3} + 2n\pi\right) = \sec\left(\frac{x}{2}\right) \quad \text{or} \quad \sec\left(\frac{4\pi}{3} + 2n\pi\right) = \sec\left(\frac{x}{2}\right)$$

$$\therefore x = \frac{4\pi}{3} + 4n\pi, \quad x = \frac{8\pi}{3} + 4n\pi$$

$$[22] \quad 2 \cos^2 \theta + 3 \sin \theta = 0, \quad 0^\circ \leq \theta \leq 360^\circ$$

$$2[1 - \sin^2 \theta] + 3 \sin \theta = 0$$

$$2 - 2 \sin^2 \theta + 3 \sin \theta = 0$$

$$2 \sin^2 \theta - 3 \sin \theta - 2 = 0$$

$$(2 \sin \theta + 1)(\sin \theta - 2) = 0$$

$$2 \sin \theta + 1 = 0$$

OR

$$\sin \theta - 2 = 0$$

$$\sin \theta = -\frac{1}{2}$$

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$$\theta = -30 + 360n = 330^\circ + 360n$$

OR

$$\theta = 180 + 30 + 360n = 210^\circ + 360n$$

$$\therefore \theta = 210^\circ, \quad \theta = 330^\circ$$

$$[24] \quad \cos 2\theta + \cos \theta = 0, \quad 0^\circ \leq \theta \leq 360^\circ$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ \text{ or } 300^\circ$$

or

$$\cos\theta = -1$$

$$\theta = 180^\circ$$

$$\therefore \theta = 60^\circ, 180^\circ, 300^\circ$$

$$[26] \quad 2\sin^2\left(\frac{x}{2}\right) - 3\sin\left(\frac{x}{2}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi$$

$$(2\sin\left(\frac{x}{2}\right) - 1)(\sin\left(\frac{x}{2}\right) - 1) = 0$$

$$2\sin\left(\frac{x}{2}\right) - 1 = 0$$

$$\sin\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\frac{x}{2} = \frac{\pi}{6} \text{ or } \frac{x}{2} = \frac{5\pi}{6}$$

$$x = \frac{\pi}{3} \text{ or } x = \frac{5\pi}{3}$$

or

$$\sin\left(\frac{x}{2}\right) - 1 = 0$$

$$\sin\left(\frac{x}{2}\right) = 1$$

$$\frac{x}{2} = \frac{\pi}{2}$$

$$x = \pi$$

$$\therefore x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

[28]

$$6 \sin^2 \theta + 5 \sin \theta - 6 = 0$$

$$0^\circ \leq \theta \leq 90^\circ$$

$$\sin \theta = \frac{-5 \pm \sqrt{25 + 4 \cdot 6 \cdot 6}}{12}$$

$$= \frac{-5 \pm \sqrt{169}}{12}$$

$$= \frac{-5 \pm 13}{12}$$

$$= \frac{8}{12} = \frac{2}{3} \quad \text{or} \quad \frac{-18}{12} > 1$$

 \emptyset

$$\sin \theta = \frac{2}{3}$$

$$\theta = 41.8087^\circ$$

$$\text{or } \theta = 180 - 41.8087 = 138.1913$$

$$\therefore \theta = 41.8087^\circ \text{ or } \theta = 138.1913^\circ$$

$$[30] \quad 2 \sin x = \cos 2x, \quad 0 \leq x \leq 2\pi$$

$$2 \sin x - \cos 2x = 0$$

$$2 \sin x - (1 - 2 \sin^2 x) = 0$$

$$2 \sin x - 1 + 2 \sin^2 x = 0$$

$$2 \sin^2 x + 2 \sin x - 1 = 0$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 + \sqrt{3}}{2}, \frac{-1 - \sqrt{3}}{2}$$

$$\text{So } \sin x = \frac{-1 + \sqrt{3}}{2} \quad \text{or} \quad \sin x = \frac{-1 - \sqrt{3}}{2}$$

$$\Rightarrow \boxed{x = 0.3947} \quad \text{or} \quad \emptyset$$

$$[36] \quad \sin x + \cos x = 1, \quad 0 \leq x \leq 2\pi$$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$$

$$2\sin x \cos x = 0$$

$$\sin 2x = 0$$

$$2x = 0 \quad \text{or} \quad 2x = 2\pi$$

$$x = 0, x = \pi$$

$$\sin 0 + \cos 0 = 1 \quad \checkmark$$

$$\sin \pi + \cos \pi = -1 \quad \text{NO}$$

$$\therefore x = 0$$

$$[38] \quad \tan x - \sec x = 1, \quad 0 \leq x \leq 2\pi$$

$$\tan^2 x - 2\tan x \sec x + \sec^2 x = 1$$

$$\tan^2 x - 2\tan x \sec x + \sec^2 x - 1 = 0$$

$$2\tan^2 x - 2\tan x \sec x = 0$$

$$\tan x (\tan x - \sec x) = 0$$

$$\tan x = 0$$

OR

$$\tan x - \sec x = 0$$

$$\frac{\sin x}{\cos x} = 0$$

$$\tan x = \sec x$$

$$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\sin x = 0$$

$$\sin x = 1$$

$$x = 0, \pi, 2\pi$$

$$x = \frac{\pi}{2}$$

$$\tan 0 - \sec 0 = 0 - 1 \neq 1$$

$$\tan \frac{\pi}{2} - \sec \frac{\pi}{2} = \text{UNDEF}$$

$$\tan \pi - \sec \pi = 0 - (-1) = 1 \quad \checkmark$$

$$\therefore x = \pi$$